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# A Generic Fuzzy Metric for Damage Recognition in Structural Health Monitoring Systems

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**Abstract** - All engineered systems will deteriorate over time due to disturbing factors such as environment, increasingly surpassing (or closely surpassing) its design capacity, subsystem (subcomponent) degradation etc. We introduce an approach for assessing and recognizing potential damage of systems by using a generic metric with fuzzy sets. We propose transforming sparse data in a Bayesian updating scheme into fuzzy sets to be used in a similarity metric to identify the health or damage state. Two case studies using different damage features are used as exemplary applications for the proposed approach.

**Keywords:** Structural Health Monitoring, Damage Metrics, Fuzzy Set Theory, Bayesian Updating

## 1 Introduction

The emerging field of Structural Health Monitoring (SHM) addresses the in-situ behavior of structures by assessing their performance and recognizing damage or deterioration. SHM involves System State Definition, Data Acquisition, Data Communication, Data Filtration, Data Reduction, Feature Extraction, Pattern Recognition and Decision Making. Each of these components is equally important to determine the state of health of a structure. Extensive research on SHM has been developed over the last decade on data acquisition, feature extraction and data reduction techniques [1-4]. However, little research has been devoted to the pattern recognition and decision making components in SHM systems.

Most of the research that examines damage recognition for SHM is focused on statistical pattern recognition techniques. Such approaches as establishing statistical process control [5], defining confidence intervals for damage indices [6], or artificial intelligence techniques like support vector classifiers [7] have been addressed in the literature. Since statistical pattern recognition can account for only distinct damage features for which data can have a crisp point-wise quantification, such approach immediately restricts damage detection to the limitations of probabilistic assumptions.

In providing damage recognition for systems structures “or systems”, two questions need to be answered: “*Is there*

*damage in the structure?*” and, if yes, “*How severe is this damage?*” The problem is that damage can occur at various rates: slowly, (environmental and time-dependent conditions), quickly and predictably, (heavy traffic loading), and quickly and unpredictably (earthquake acceleration). Damage can also occur at a variety of non-distinct and overlapping levels of severity such as minor, moderate, severe and very severe.

Observation of most features suggested in the literature for damage identification (*usually named damage index*) showed that a major challenge in damage recognition in SHM is the non-stationary sample space of the damage index; it is dependent on the health state of what is assumed to be a healthy system. It was also observed that the frequency of occurrence of a healthy state in the database is not constant from one set of observations to another. Therefore, it becomes obvious that damage cannot be classified as a random process and probabilistic assumptions for damage recognition might not hold. It can also be argued that most suggested probabilistic based damage recognition techniques represent a one-dimensional approach from the uncertainty point of view being limited to a smaller scope of information (*statistical knowledge*). However, given the relatively low sampling rates in SHM systems, the scarcity of historical observations to form expert knowledge, and finally the necessary health (or damage) patterns overlapping to represent interconnected states, pattern uncertainty might have a significant effect on decision making for SHM systems. However, we assert that uncertainty in damage recognition is more epistemic (subjective) than aleatoric (objective), and that is we can improve our knowledge about damage and reduce uncertainty of its recognition as we obtain further observations. The major difference between aleatoric and epistemic uncertainties is that aleatoric refers to random uncertainties that are irreducible while epistemic uncertainty is related to system knowledge and thus is a reducible uncertainty [8, 9].

## 2 A Generic Fuzzy Damage Recognition Approach

Given the inherent uncertainty in the damage features (indices) and admitting a level of imprecision in damage states using fuzzy sets, we can feasibly define damage levels. We propose a new method to determine damage levels by establishing fuzzy sets on the damage index domain. In doing this, we combine sparse measured point data and interval data, Bayesian updating, and expert judgment within the construct of the fuzzy logic framework to substantiate fuzzy sets for damage recognition. We will explain first our approach to establish the health patterns and a method for providing damage recognition thereafter.

### 2.1 Establishing fuzzy health or “damage” patterns

Most damage detection approaches produce a damage index. If the damage index (*we will denote it  $\lambda$* ) is monitored during a period of healthy performance  $T_{\text{Healthy}}$ , a set of consecutive values of this index can be acquired. We suggest using this set of consecutive values to produce a group of fuzzy membership functions that describe all damage levels. The state of the structure will then be determined based on the vector’s (fuzzy pattern) degree of similarity to the defined fuzzy damage levels. This current approach expands on the all-probabilistic method in most existing damage identification methods e.g. [10] and [11].

The disadvantage of the all-probabilistic approach is that it is being constrained with many assumptions. One assumption is the assignment of a symmetric distribution to the response levels. Without an infinitely number of measurements, this cannot be proven to be the case. Where the luxury of having an infinitely number of measurements or performing extensive computer simulation is not feasible, pragmatic damage recognition where existing information is used as prudently as possible becomes the answer. This means that we will use measured data along with expert judgment to modify the fuzzy membership functions using both dense and sparse data. We will have ample data during the healthy training period during which other methods of system performance assessment might be deployed. Hence we will develop a “Healthy” fuzzy set that reflects an undamaged structural state. The shape of this fuzzy set can be determined according to the observed values of the damage index during healthy performance. This can be assumed to be Gaussian (or any other shape of membership function until enough observations are acquired). The problem is that all available observations represent only “Healthy” performance; successful damage recognition requires fuzzy patterns (*membership functions*) to describe possible damage levels. To accomplish building the remaining fuzzy patterns, we suggest adopting the approach proposed by Huyse and Thacker [12] to overcome the challenge of risk assessment under

conflicting and insufficient data. We, therefore, suggest developing fuzzy membership functions that mimic the shape of a probability distribution that is suitable for sparse data namely Poisson distribution function.

The Poisson distribution is useful for cases where infrequent events are seen; it essentially describes a counting process. We are taking a conceptual departure from this and using only the shape of the Poisson distribution over a continuous domain of expected values as in [12]. In our application, infrequent and high damage index “ $\lambda$ ” values might be measured and the shape of the distribution (*membership function*) can be continually updated. This summarizes the data frequency and gives us a well justified value on which to define our damage fuzzy membership functions. This proposed approach is much more convenient in that it solely depends on inference of healthy observations with expert judgment without demanding any further knowledge via structural simulation or special field testing. Here, for a generic approach, four structural health patterns (damage levels) ranging from healthy to significantly damaged are proposed. The non-distinct boundaries between these health patterns and the inherent overlap therewith make fuzzy systems a suitable candidate for damage recognition. We begin by defining the “Healthy” pattern given the fact that a reasonable amount of observations are made during a time period,  $T_{\text{Healthy}}$ . Therefore, the membership function that describes the fuzzy set “Healthy” is defined as a left-shouldered fuzzy set using the Gaussian function described in equation (1) in a similar fashion to damage analysis for earthquakes [8]:

$$\mu_H(x) = \begin{cases} \exp\left(\frac{-(x - \bar{\lambda}_H)}{\sigma_{\lambda_H}^2}\right) & x \geq \bar{\lambda}_H \\ 1 & x < \bar{\lambda}_H \end{cases} \quad (1)$$

$\mu_H(x)$  represents the membership function of the fuzzy set representing healthy pattern that has an average observed damage index of  $\bar{\lambda}_H$  and a spread of  $\sigma_{\lambda_H}$ . Furthermore, information from this first fuzzy set can be used to develop the proximate fuzzy set, “Little Damage”.

It is important to note that the Gaussian shape is used only for simplicity of calculation and the use of other left-shouldered membership function shapes for the healthy fuzzy set shall not affect the validity of the approach. We thus, begin with an uncertain knowledge about the domain “Little Damage”. What is certainly known is that our universe of discourse is the set of all possible  $\lambda$  values, non-negative and real numbers and that all accessible information has been attained during healthy performance period. Thus we expand the universe of discourse to incorporate the fuzzy set “Little Damage”. Since the domain of this first set, “Healthy” is known, the lower

bound of “Little Damage” can be located by assuming it to match the mean healthy value  $\bar{\lambda}_H$  such that the universe of discourse of the fuzzy set “Little Damage” denoted as  $LD$  can be defined as:

$$LD = \{x/x \geq \bar{\lambda}_H\} \quad (2)$$

The following step is to identify the shape of the membership function for  $LD$ . An approach to describe the fuzzy set “Little Damage” given the limited knowledge about the health or (damage) index  $\lambda$  in this domain is to assign the fuzzy set  $LD$  a non-informative prior distribution. Such distribution attempts to represent a certain level of initial lack of knowledge about the system [9]. Unless specific knowledge about the system is available, a uniform distribution is traditionally assigned to describe the non-informative prior. In our case of damage recognition using the  $\lambda$  values, we realize that the fuzzy sets are constructed in such a way that each fuzzy set will cover its own range of  $\lambda$  values while reserving an inherent overlap with the other fuzzy sets. This knowledge negates the possibility of using uniform distribution to represent the non-informative prior. Therefore, Jeffrey’s non-informative prior [9] distribution over the domain  $X$  ( $\lambda$  values) that assumes no-observation,  $x$ , is used here:

$$f(X) = \frac{1}{\sqrt{X}} \quad (3)$$

Therefore, we will calculate the degree to which data  $x$  is contained in the domain  $X$  using Bayesian updating. Bayesian Updating is a natural consequence of Bayes’ Theorem. Bayes’ Theorem simply combines prior knowledge about a parameter with additional support data to compute the subsequent knowledge of the parameter [8,9]. This updated knowledge is known as the posterior distribution  $f(x/X)$  and is proportional to the product of the likelihood  $\ell(X/x)$  and the prior distribution  $f(X)$  as:

$$f(x/X) = \frac{\ell(X/x) \cdot f(X)}{\int \ell(X/x) \cdot f(X)} \quad (4)$$

Our approach goes a step further in accommodating uncertainty by using interval data  $[x_1 \text{ to } x_2]$ . In doing so we will need to compute the likelihood that the damage domain  $X$  contains the observation interval data  $[x_1 \text{ to } x_2]$  which can be computed as

$$\ell(X/[x_1, x_2]) = \int_{x_1}^{x_2} f(x/X) dx \quad (5)$$

We will assume that the likelihood of damage to follow Poisson’s density function. This is based on the

assumption that damage events represent non-frequent occurrences which harmonizes with the typical use of Poisson distribution to represent non-frequent and independent variables [9]. Poisson distribution is expressed as

$$f(x/X) = \frac{X^x}{x!} \exp(-X) \quad (6)$$

Substituting equation (6) into equation (5) yields the likelihood given interval data as

$$\ell(X/[x_1, x_2]) = \exp(-X) \sum_{x=x_1}^{x_2} \frac{X^x}{x!} \quad (7)$$

Bayesian updating to produce the first estimate of the membership function that describes the fuzzy set “Little Damage” can be performed by substituting equations (6) representing the likelihood of damage and equation (3) representing the non-informative prior into equation (4) which yields

$$f_1([x_1, x_2]_1/X) = \frac{\exp(-X) \sum_{x=x_1}^{x_2} \frac{X^{x-1/2}}{x!}}{\sum_{x=x_1}^{x_2} \frac{\Gamma\left(x + \frac{1}{2}\right)}{x!}} \quad (8)$$

This first estimate assumes one single interval observation with  $x_1$  being the highest  $\lambda$  healthy measurement and  $x_2$  defined as an expert seeded point. These will be updated using subsequent interval observations to supply the membership function that describes the fuzzy set “Little Damage” for the  $i^{\text{th}}$  interval observation  $[x_1 \text{ to } x_2]_i$  as:

$$f_i([x_1, x_2]_i/X) = \frac{\exp(-X) \sum_{x=x_1}^{x_2} \frac{X^x}{x!} * f_{i-1}}{\sum_{x=x_1}^{x_2} \frac{\Gamma(x)}{x!}} \quad (9)$$

A similar procedure is used to develop initial estimates for the other damage fuzzy sets in accordance with our knowledge of fuzzy set boundaries. Additionally, the last fuzzy set, “Significant Damage”, will be assumed to act as the upper bound on the universe of discourse much like “Healthy” such that it is right-shouldered as

$$\mu_S(x) = \begin{cases} Eq.(8-9) & x < \lambda_S^* \\ 1 & x \geq \lambda_S^* \end{cases} \quad (10)$$

where,  $\lambda_S^*$  is the prototypical “Significant Damage”  $\lambda$  value.

To summarize,  $\lambda$  values over a time period will be used to develop the fuzzy set, “Healthy”. In succession, the lower bounds for the remaining fuzzy sets will use the shape of Jeffrey’s non-informative prior in posterior updating of the likelihood represented by Poisson distribution. Interval data within the bounds set forth by experts will update the functions in equations (8), (9) and (10) to form three fuzzy damage sets, “Little Damage” “Moderate Damage” and “Significant Damage”. This is an original process that incorporates both single point and interval data to build on received evidence in the form of statistical data or expert judgment. The approach is also generic that it can be applied regardless of the damage feature and the shape of the membership functions to describe the “Healthy” fuzzy set. The developed fuzzy sets are ready to be used to recognize recent observations at time of unknown health state.

## 2.2 Fuzzy damage recognition

Now we want to recognize (identify or classify) a set of consecutive input observations ( $\lambda$ ) into one of the pre-defined fuzzy set levels of damage. This recognition requires a concept in fuzzy logic, degree of similarity. The degree of similarity of an input observation vector acquired at any time period N will be measured and compared to the established fuzzy sets in order to determine the structure’s health or (damage) state. Following Ross [4] we introduce a damage metric ( $DM_Y$ ) that represents the degree of similarity of any newly observed fuzzy set of observations  $\lambda_{New}$  and the four existing fuzzy sets “Healthy”, “Little Damage”, “Moderate Damage” and “Significant Damage”, here denoted as  $Y$ . The fuzzy damage metric can be determined

$$DM_i = \frac{1}{2} [(\lambda_{New} \bullet Y_i) + (\lambda_{New} \oplus Y_i)] \quad (11)$$

where  $(\lambda_{New} \bullet Y_i)$  represents the inner product of the two fuzzy vectors and  $(\lambda_{New} \oplus Y_i)$  represents the outer product of the two vectors [4]. The similarity metric “ $DM_i$ ” is thus computed to represent the degree of similarity between the new vector of  $\lambda$  observations representing the unknown structural health and represented by the fuzzy set  $\lambda_{New}$  and each of the four fuzzy structural health patterns previously defined. Using principles of the maximum approaching degree explained in [4], the fuzzy health or “damage” pattern with the maximum similarity metric “ $DM$ ” will be the closest health pattern to describe the structural health.

## 3 Case Studies

To demonstrate the versatility of the proposed model two case studies where damage recognition is required are examined. In both case studies, a feature that represents damage in the structure (namely damage index) is computed. Values of the damage index are simulated to represent the structural response during different health or damage states of the structure. The model is first used to construct all the fuzzy damage membership functions using the damage index values observed only during the healthy time. The model ability to recognize other health or damage levels using a vector of damage index observations is tested. The model results are compared to the damage state (class) as reported by the case study.

The first case study is reported by Reda Taha et al. [13] for damage identification of a model structural steel bridge modelled using a finite element (FE) model. Accelerometers are assumed to be distributed over the bridge area. Regional input and desired signals are simulated for training during healthy performance and for testing during healthy and damaged instances using finite element modeling. Accelerations of the FE model are analyzed using a neural-wavelet damage diagnosis module. The damage index ranges between 0 and 1000 and expresses energy of signals computed in the wavelet domain. The higher the damage index the more severe the level of damage in the structure. More details about the neural-wavelet damage diagnosis module and the wavelet damage feature can be found elsewhere [10]. The damage index ( $\lambda_I$ ) values for the model steel bridge observed during healthy performance are shown in Table 1. Data shown in Table 1 are used to construct the fuzzy damage sets on the universe of discourse of  $\lambda_I$ . To test the model a vector of unclassified  $\lambda_I$  observations (Table 2), known by the FE model to represent little damage in the steel bridge, are forwarded to the model for damage recognition.

Table 1. Healthy observations of  $\lambda_I$

$t^*$	1	2	3	4	5	6
$\lambda_I$	41.4	58.9	88.0	28.4	14.8	29.1
$t^*$	7	8	9	10	11	
$\lambda_I$	93.6	26.6	53.7	39.9	122.7	

$t^*$  is time instance

Table 2. Unknown health or damage set observations of  $\lambda_I$

$t^*$	1	2	3
$\lambda_I$	174.2	260.1	291.0

$t^*$  is time instance

The second case study examines failure of a composite beam-steel plate connection as reported by Salvino et al. [11]. Damage identification was performed by observing the phase of traveling structural wave in the connection. A damage index ( $\lambda_2$ ) that represents the

average phase change in the strain signals between the healthy and damage structure was reported. The original damage index reported by [11] ranges between 0 and 0.35 while ( $\lambda_2$ ) used here were scaled to range between 0 and 350. More details about the phase angle damage index can be found elsewhere [11]. The damage index ( $\lambda_2$ ) values for the composite beam-steel plate connection as simulated using FE models during healthy performance are shown in Table 3. To test the model a vector of unclassified  $\lambda_2$  observations (Table 4), known by the FE model to represent significant damage in the connection, are forwarded to the model for damage recognition.

Table 3. Healthy observations of  $\lambda_2$

$t^*$	1	2	3	4	5	6
$\lambda_2$	4	10	14	14	16	31
$t^*$	7	8	9	10	11	12
$\lambda_2$	32	36	39	42	46	50

$t^*$  is time instance

Table 4. Unknown health or damage set observations of  $\lambda_2$

$t^*$	1	2	3	4	5
$\lambda_2$	254	258	258	261	265

$t^*$  is time instance

## 4 Results and Discussions

Our damage recognition approach first produces fuzzy sets, “Healthy”, “Little Damage”, “Moderate Damage” and “Significant Damage” for the two cases as shown in Figures 1 and 2. Only the healthy observations are used to define the first “Healthy” fuzzy set. Subsequently, the extreme value from the “Healthy” observations was used with an expert-defined “seed” point. This seed point represents the expert’s opinion on what defines the various damage sets. This point becomes obsolete as more data are incorporated in the updating process.

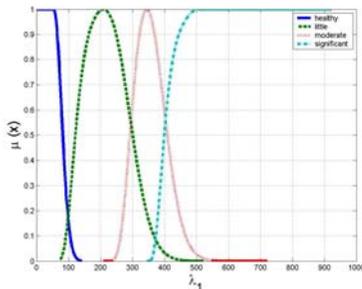


Figure 1. Final shape of the membership functions for the four damage fuzzy sets for the first case study.

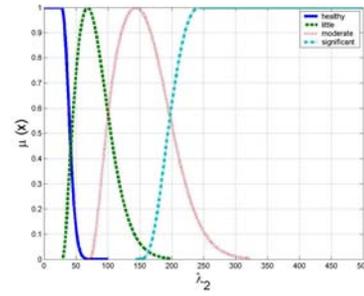


Figure 2. Final shape of the membership functions for the four damage fuzzy sets for the second case study.

Pictorial representations of the unknown fuzzy set representing the unknown observations for both case studies are shown in Figures 3 and 4 for the first and second case studies respectively. Equation (10) was then used to recognize the observation sets as one of these four fuzzy damage sets for both case studies. The similarity metrics ( $DM_1, DM_2, DM_3, DM_4$ ) of the testing instances with respect to the four structural health fuzzy sets (Healthy, Little Damage, Medium Damage and Significant Damage) for both case studies were computed and are presented in Table 5.

Our method identifies the structural response of the first case study as “Little Damage” with a slight tendency to “Moderate Damage.” Similarly, the structural response of the second case study is identified as “Significant Damage” with some overlap into “Moderate Damage.” Thus, it can be observed that the fuzzy damage recognition algorithm was capable of detecting the damage level in both structures very successfully. Additionally, we can get a clear picture of the overall state of the structure by knowing the amount of its identified state and its relation to a nearby damage level. Further validation of the model is currently being considered.

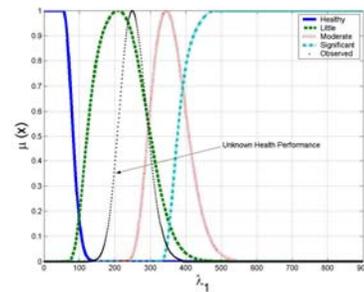


Figure 3. Final shape of the membership functions along with the unknown damage set for the first case study

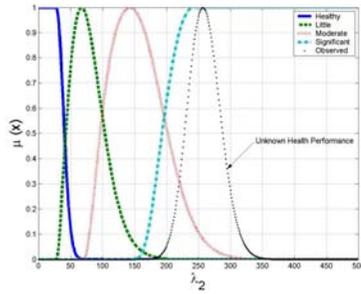


Figure 4. Final shape of the membership functions along with the unknown damage set for the second case study

Table 5. Fuzzy Damage Metric for the two case studies

	$DM_1$	$DM_2$	$DM_3$	$DM_4$
$\lambda_1$	0.50	0.98	0.76	0.53
$\lambda_2$	0.50	0.50	0.65	1.00

## 5 Conclusions

We have demonstrated a method to quantify evidence of damage levels by means of the computations of fuzzy set theory. Moreover, we have shown that this method and metric are generic for any structural health monitoring system regardless of the damage feature. The proposed method was used to construct fuzzy health (or damage) patterns for two case studies using healthy observation data. Furthermore, our method used Jeffery's non-informative prior with expert "seed" data in a Bayesian updating scheme. Subsequently, a similarity metric was used to identify any new set of observations into a particular level of damage. This generic method defines the location and shape of the membership functions using statistical based techniques. Thus it becomes an approach with far-reaching potential as it is independent of the feature used to represent damage. The method was proven to be both accurate and versatile.

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