Development of Analytical and Experimental Dynamic Model Representations for the Prediction of Baseball Bat Performance

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Dynamics Summer School

Outline

- Introduction
- Analytical Models
- Experimental Testing
- Model Comparison
- Reduction/Expansion
- Simulation Case
- Conclusions

Motivation

- Baseball bats are regulated in an effort to control game play characteristics and to ensure player safety
- Current methods require large equipment setups or destructive methods to test for bat alterations
- A new method is herein proposed by which to verify bat performance characteristics

Methodology

Detailed Analytical Models

[Mn,Kn] ⇒ [Shapes,Frequencies]

SEREP or
HYBRID
Transformation

Reduction

Reduced Model [Ma,Ka]

Comparison

Experimental Model

$[\Omega^2, E]$

Estimates for
Stiffness and Mass
Matrices:

$[E^g]^T[I]\,[E^g]=[Me]$

$[E^g]^T[\Omega^2]\,[E^g]=[Ke]$

Analytical Models

Detailed Analytical Models

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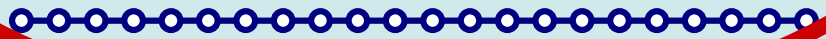

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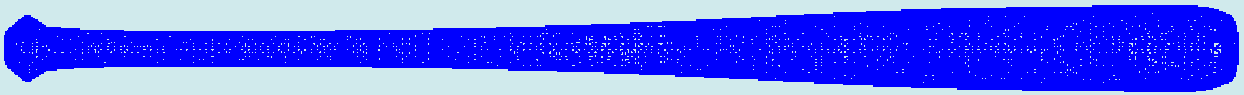
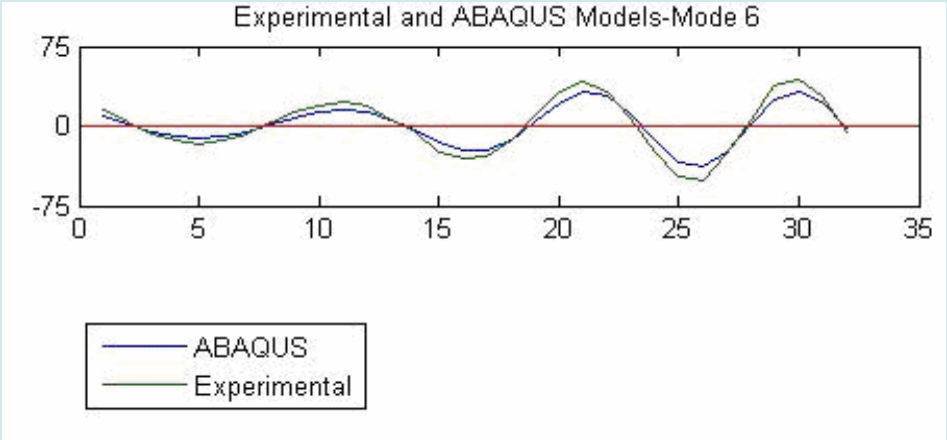
Analytical Models: FEM

- Over 15,000 nodes and 70,000 solid linear tetrahedral elements
- Assumed isotropic properties
- Frequencies and mode shapes (based on nodes on outer profile) extracted



Analytical Models: Beam Model

- Smaller model with simpler mass and stiffness matrices, modeled at one half inch intervals
- Assumed isotropic properties
- Compared to finite element model for accuracy and then reduced for comparison to experimental model



Experimental Analysis

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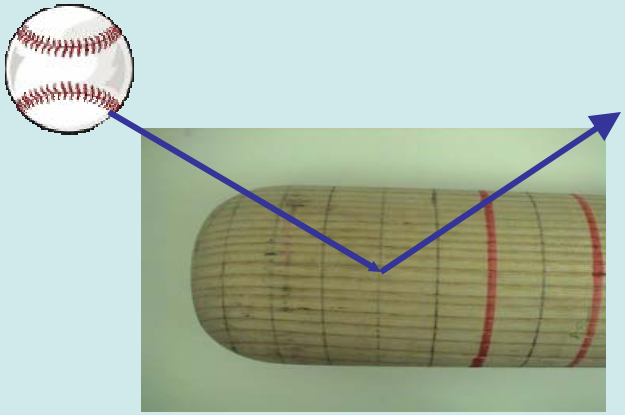
Testing Procedure

- The wood bat was suspended from each end by surgical tubing to simulate a free-free boundary condition.
- Two accelerometers were placed orthogonal to each other at one inch from the large end of the bat.

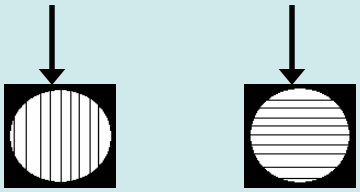
Testing Procedure (cont.)

- The bat was impacted at one inch intervals both parallel and perpendicular to the wood grain.
- Both in plane and out of plane responses were measured to assure relatively straight impacts
- The response was recorded in frequency response form and imported into MEScope for evaluation.

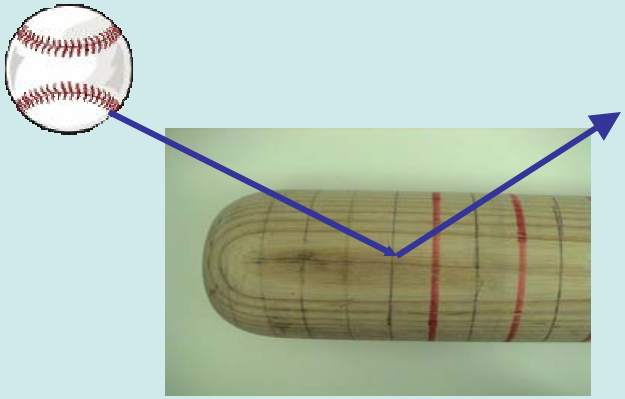
Experimental Results: Frequency Response



With Grain (X-dir.)

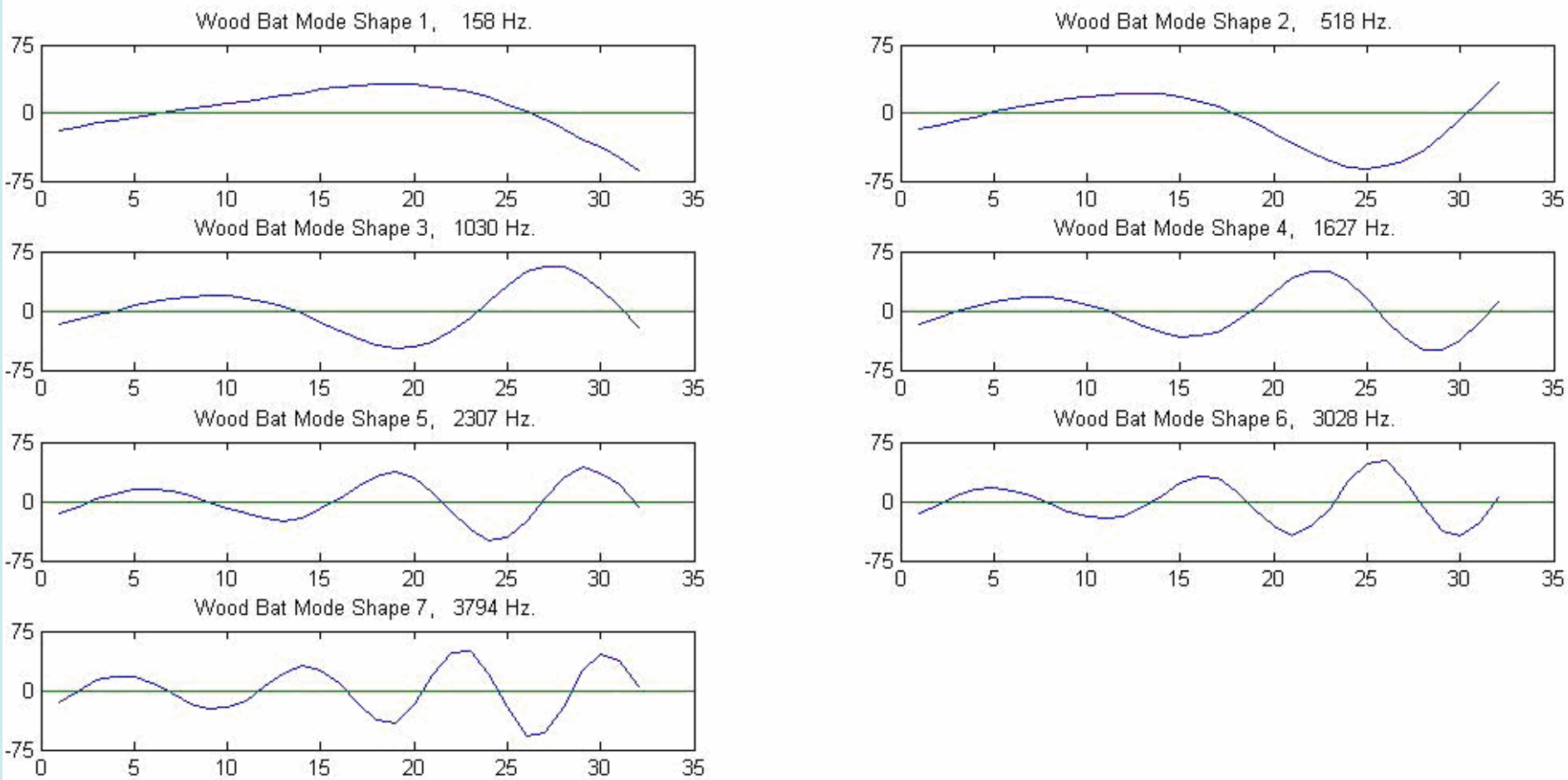


X-dir. (Hz.)	Y-dir. (Hz.)
157.7	159.7
517.7	525.6
1030	1043
1626	1659
2303	2361
3019	3113
3787	3905

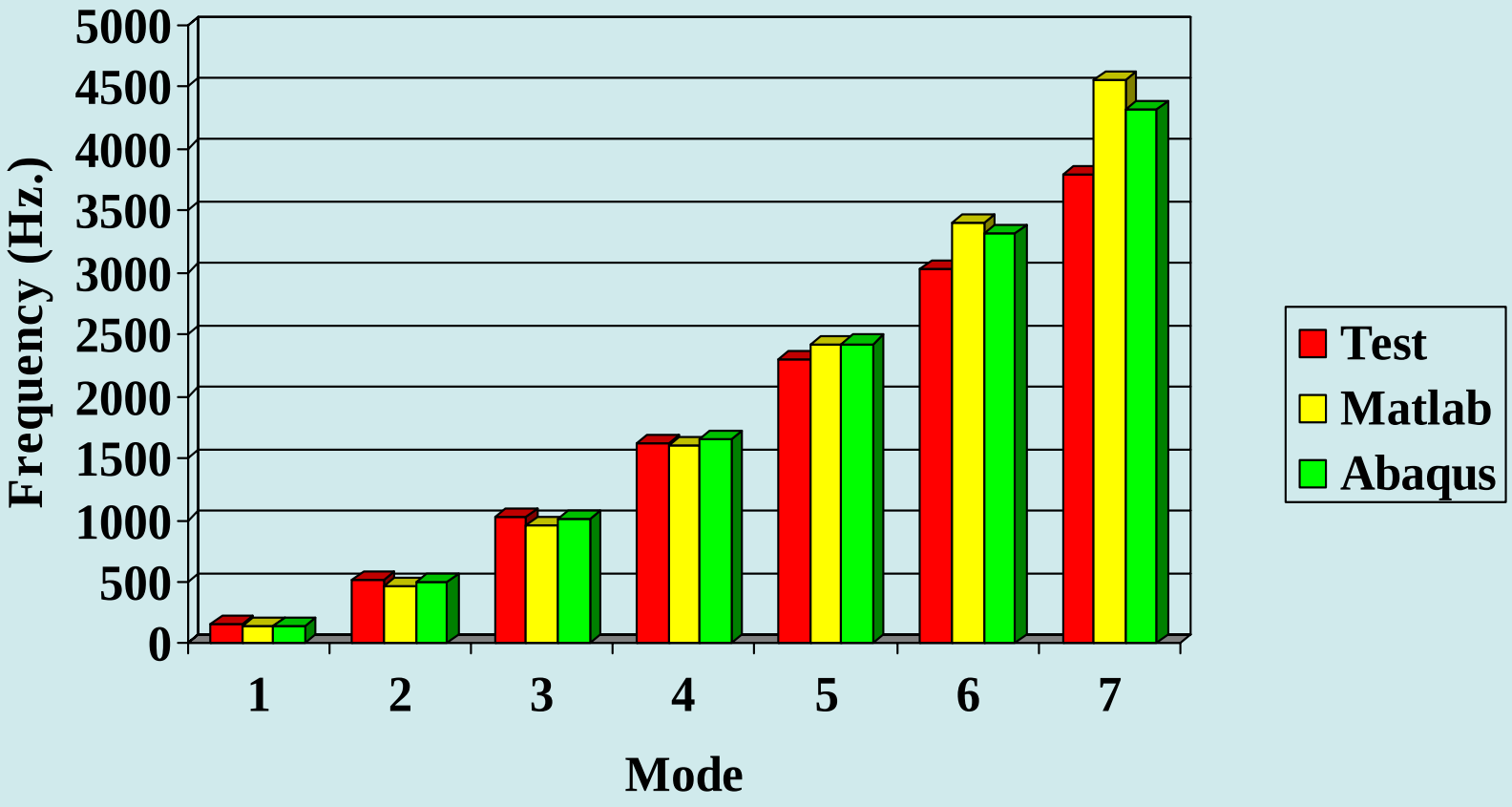


Against Grain (Y-dir.)

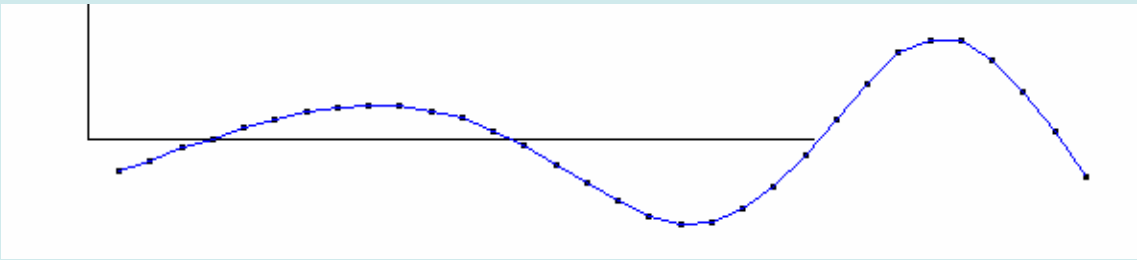
Experimental Results: Mode Shapes

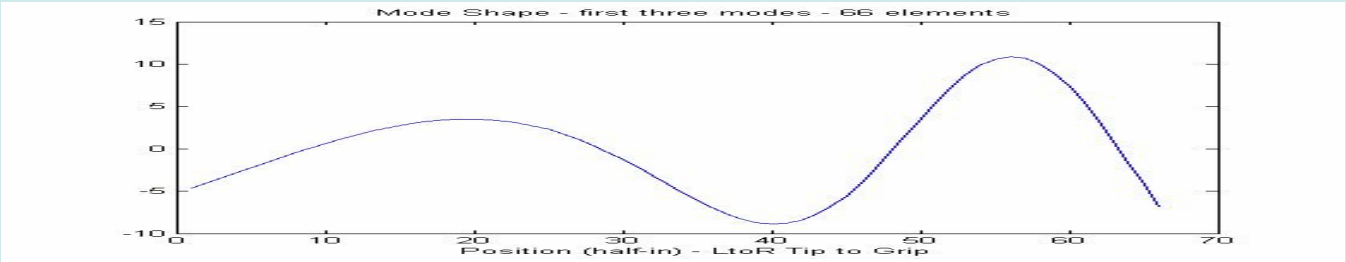


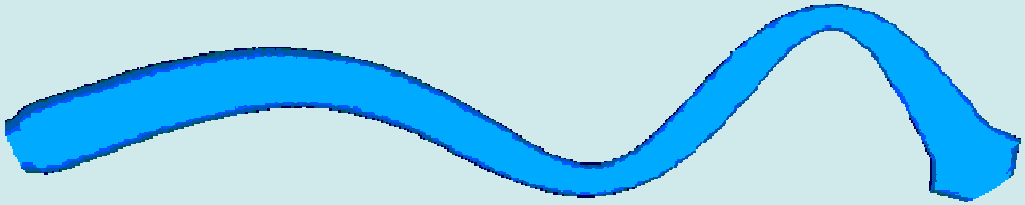
Model Comparison: Frequencies



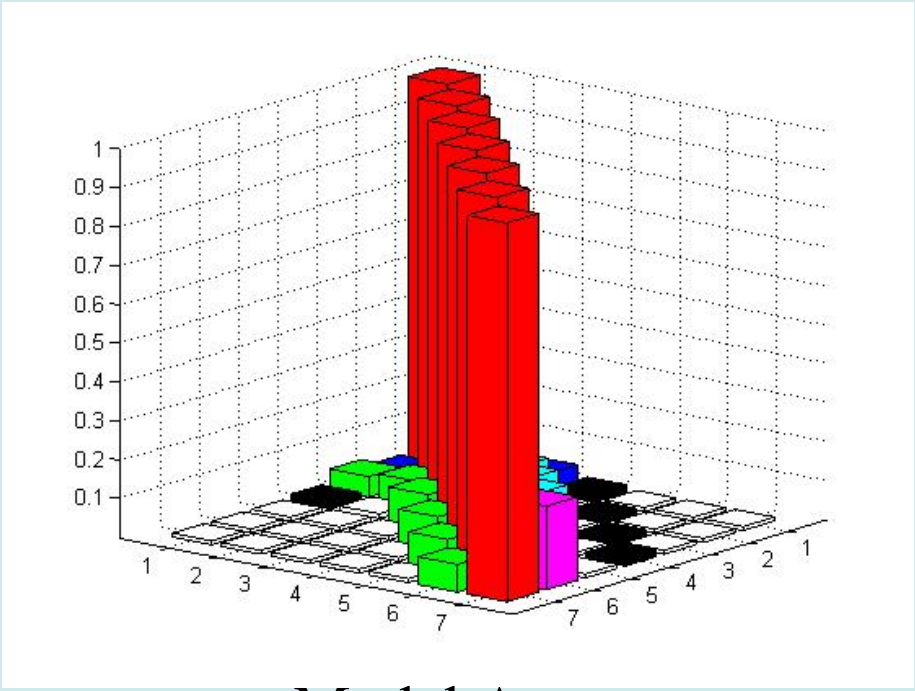
Model Comparison: Mode Shape Similarity



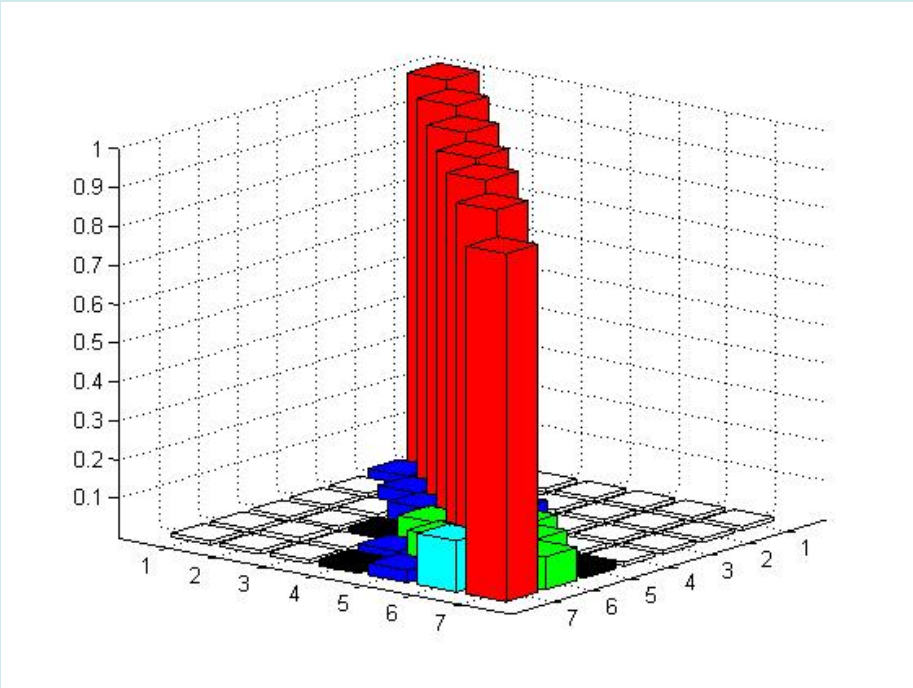




Model Comparison: Beam Model Compared to Finite Element Model

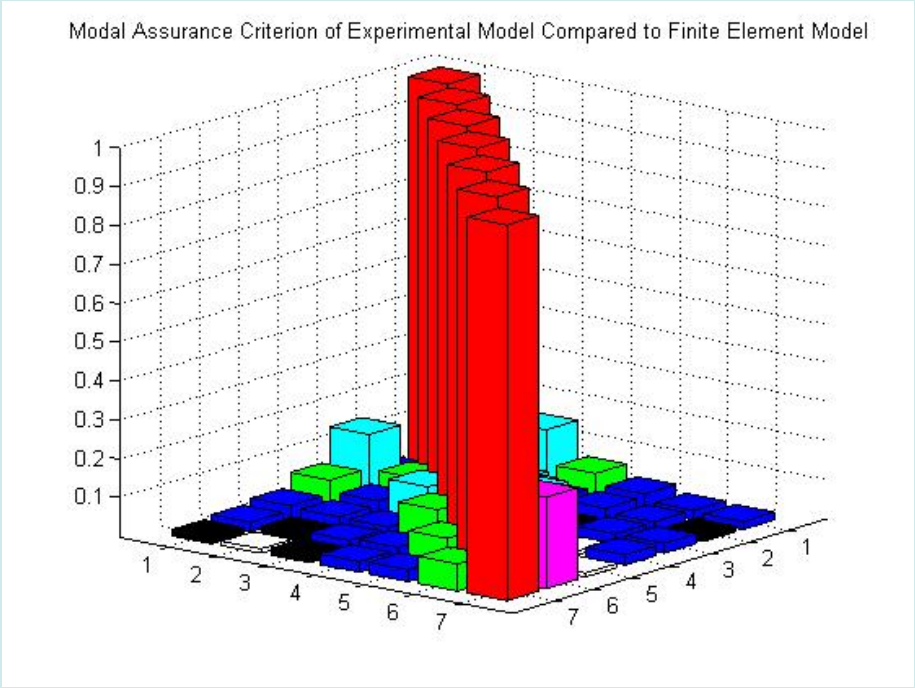


Modal Assurance Criterion (MAC)

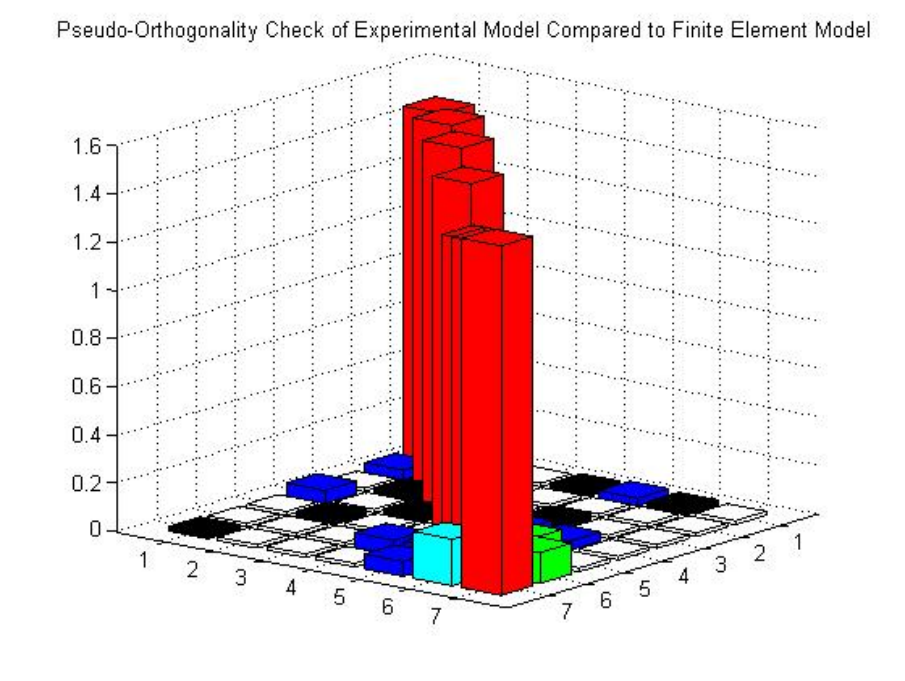


Pseudo-Orthogonality Check (POC)

Model Comparison: Experimental Model Compared to Finite Element Model

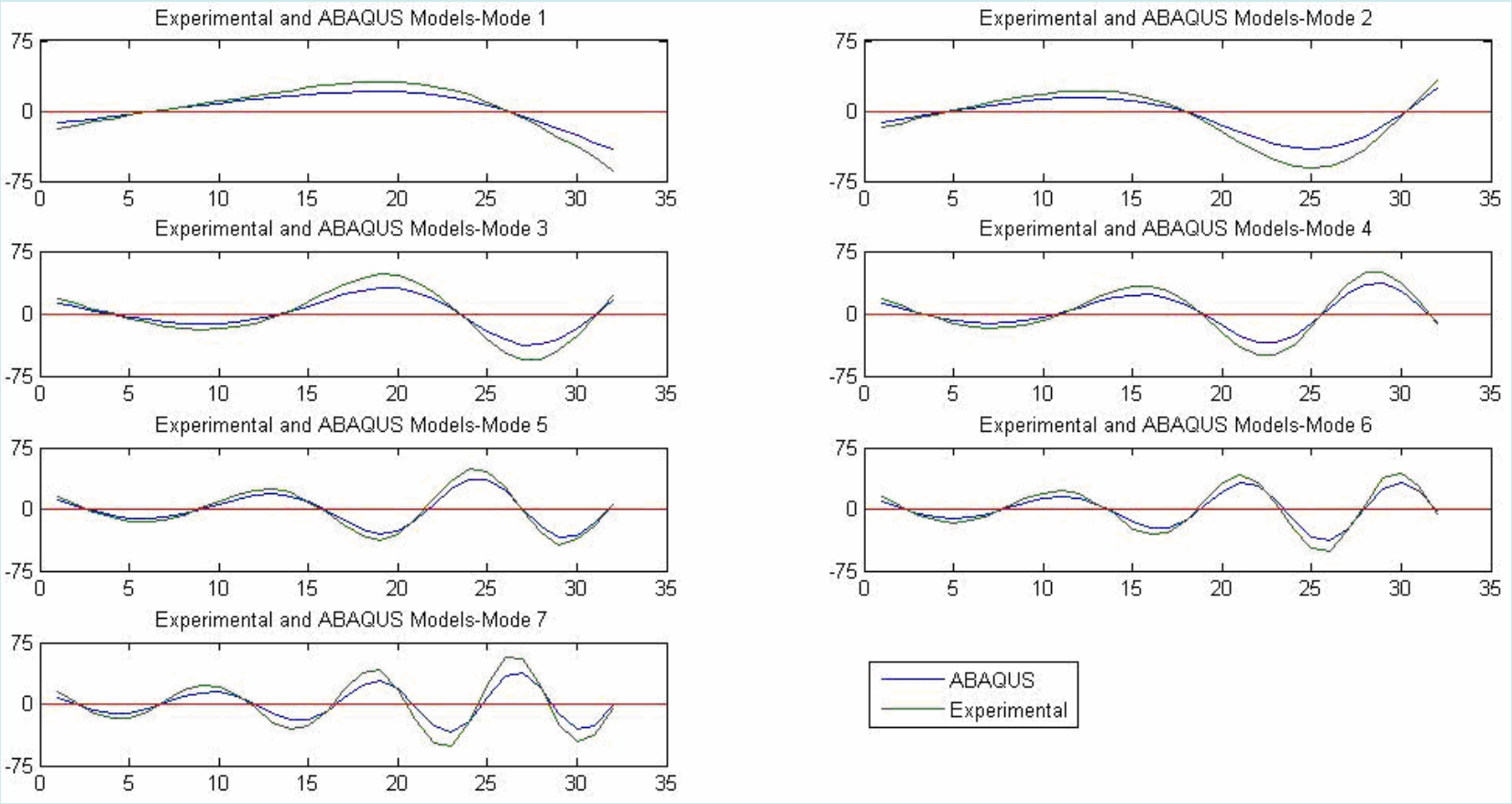


MAC

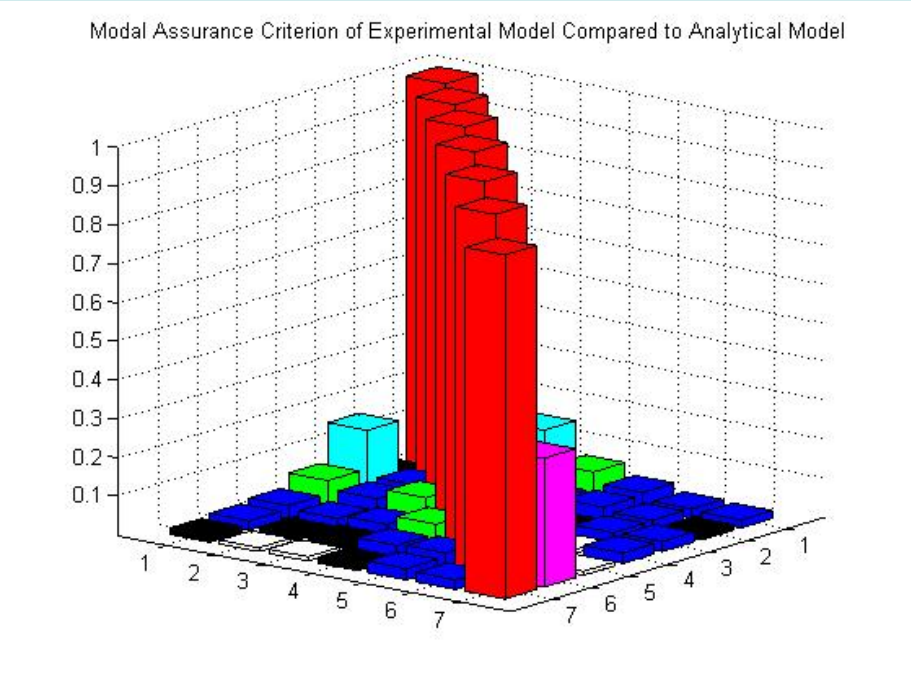


POC

Model Comparison: Experimental Model Compared to Finite Element Model (cont.)



Model Comparison: Experimental Model Compared to Beam Model



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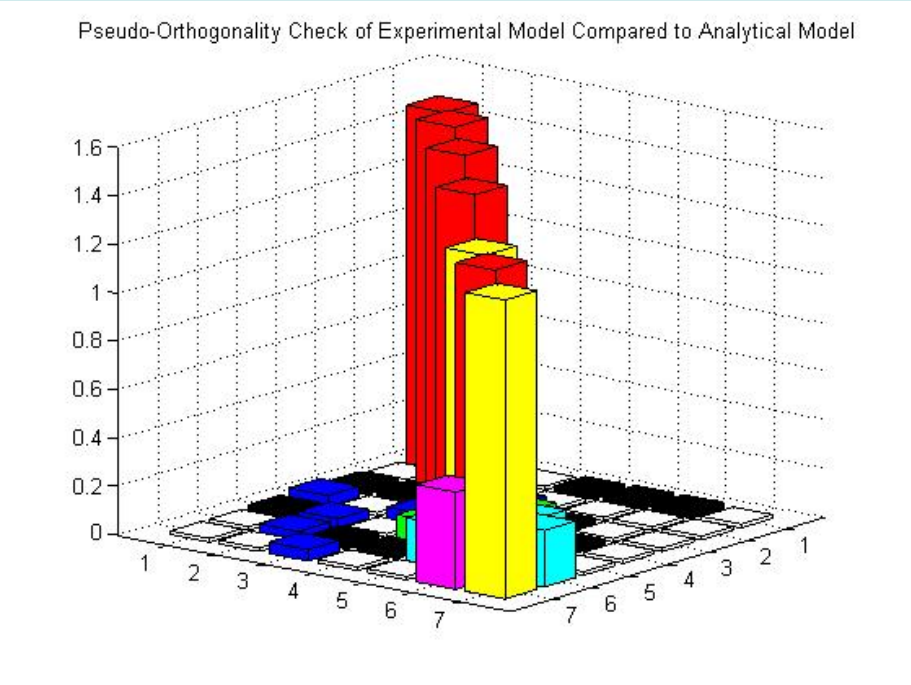
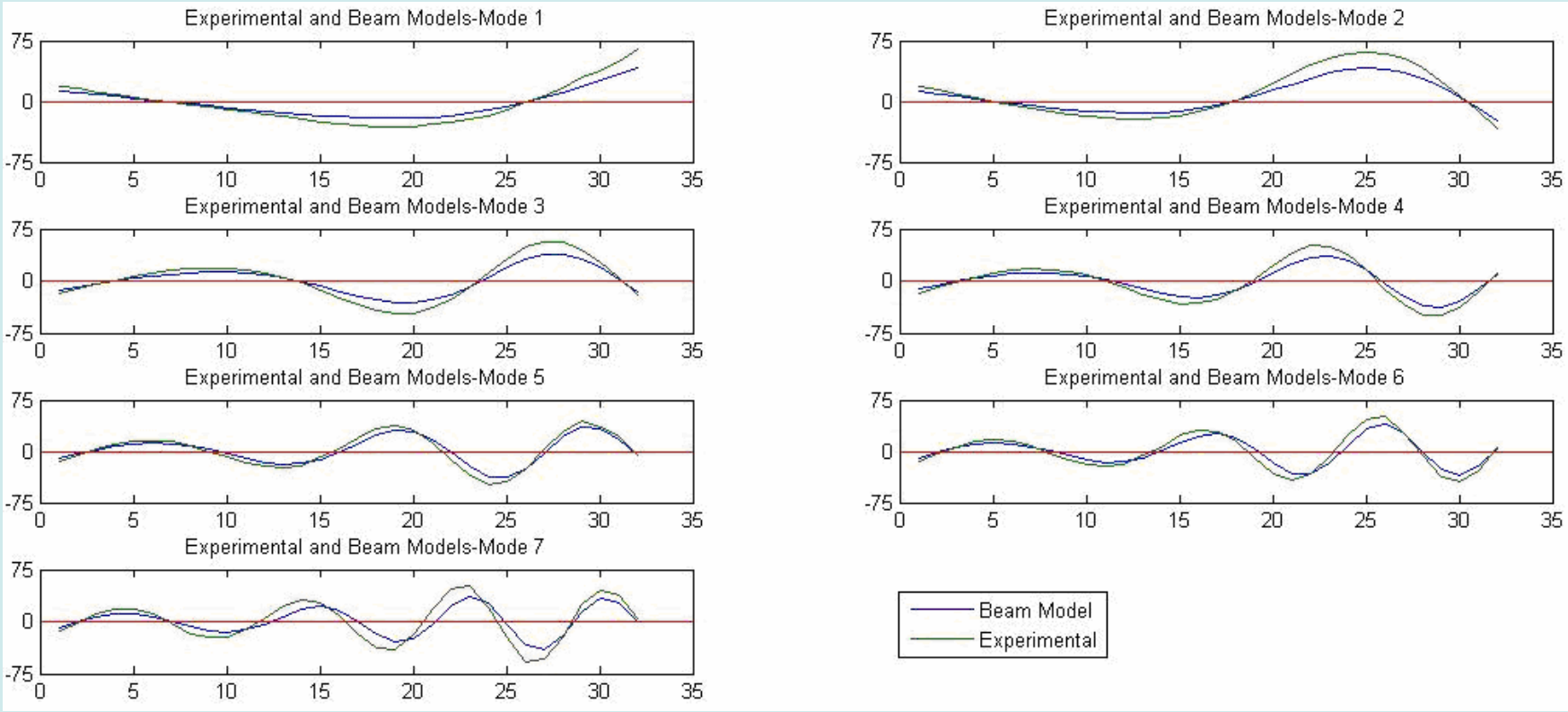


POC

Model Comparison: Experimental Model Compared to Beam Model (cont.)



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Comparison

The goal is to create an analytical model with fewer degrees of freedom that maintains modal properties

Large Model: Number of Degrees of freedom = n

Number of Modes = n

Reduced Model: Number of Degrees of freedom = a < n

Number of Modes = m $\leq$ a < n

Reduction Methods: SEREP and HYBRID

Model Reduction

$$[M_a] = [T]^T [M_n][T]$$

$$[K_a] = [T]^T [K_n][T]$$

Guyan*

$$[T_s] = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix}$$

*Static representation

SEREP*

$$[T_U] = [U_n][U_a]^g$$

*SEREP maintains exact frequencies and mode shapes

Problem with SEREP when DOF>MODES

Issue: $\underset{[nxm]}{[U_n]}\underset{[mxa]}{[U_a]^g} \Rightarrow \underset{[nxa]}{[T_U]} \Rightarrow$ Rank Deficient Matrices

Hybrid:

Solution: $[T_H] = [T_U] + [T_S][[I] - [U_a][U_a]^g]$

Experimental Mass and Stiffness

Modal Mass and Stiffness Matrix Calculation:

$$[E]^T[M][E] = [\overline{M}] = [I]$$

$$[E]^T[K][E] = [\overline{K}] = \lfloor \Omega^2 \rfloor$$

Generalized Inverse Equations to Calculate Mass and Stiffness from Experimental Data:

$$[E^g]^T[I][E^g] = [M]$$

$$[E^g]^T[\Omega^2][E^g] = [K]$$

Matrix Improvement Method (AMI)

Seeding the inverse calculation with physical characteristics enables a better comparison with the reduced analytical matrices.

$$[M_I] = [M_S] + [V]^T \left[I - \overline{M}_S \right] [V]$$

$$[K_I] = [K_S] + [V]^T \left[`\omega^2 + \overline{K}_S \right] [V] - \left[[K_S][E][V] \right] - \left[[K_S][E][V] \right]^T$$

Where:

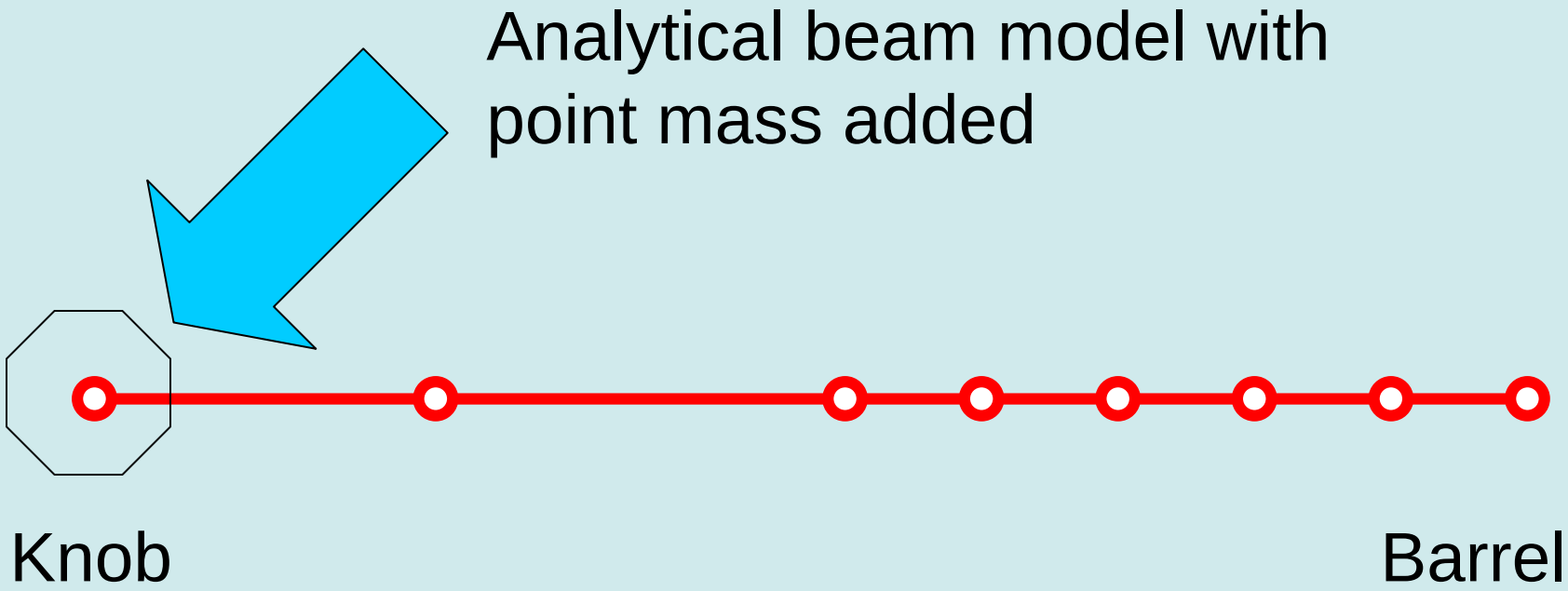
$$[V] = \left[[E]^T [M][E] \right]^{-1} [E]^T [M] = \left[\overline{M} \right]^{-1} [E]^T [M]$$

(P. Avitabile, 2003)

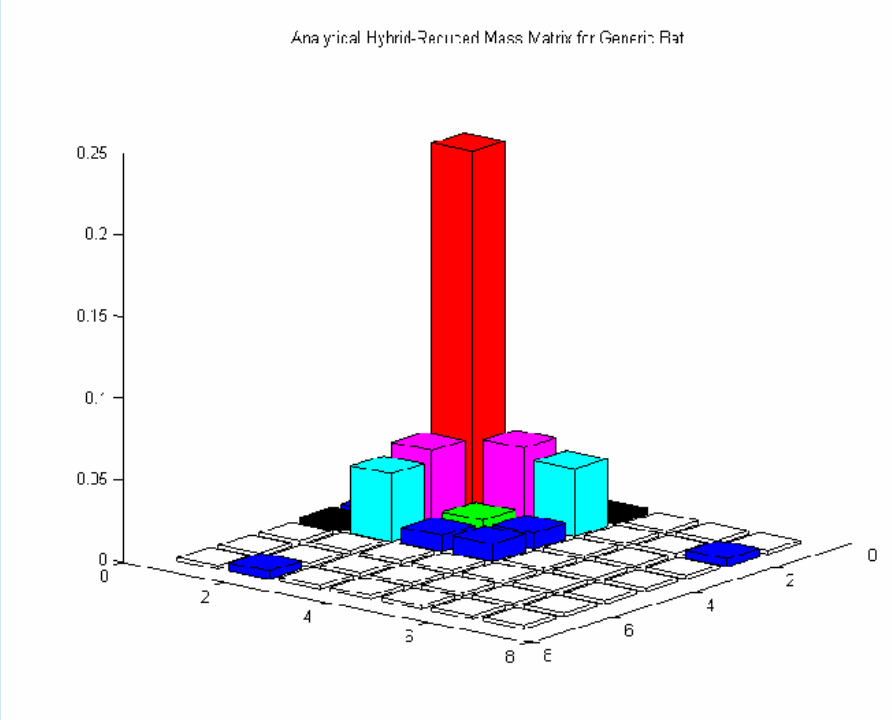
Method Validation

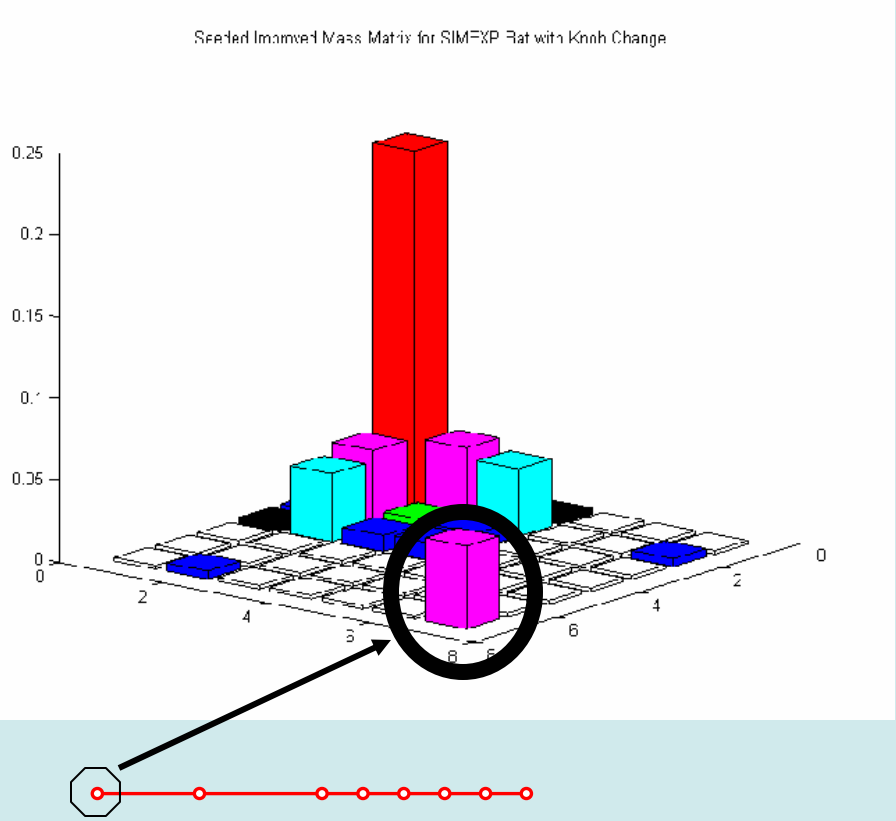
- A simulated case was designed
  - Purpose:
    - Determination of the utility of the proposed method
  - Requirements:
    - Ease of comparability to original analytical model
    - Ability to represent a possible physical manipulation of a real bat

Simulated Experimental Case



Reduced Mass Matrices

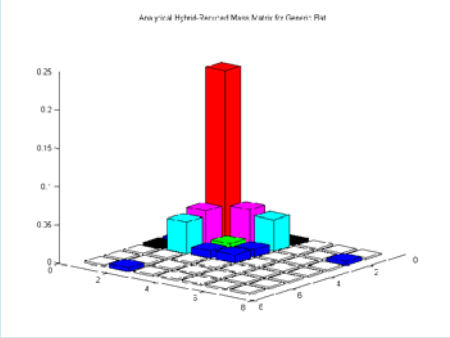




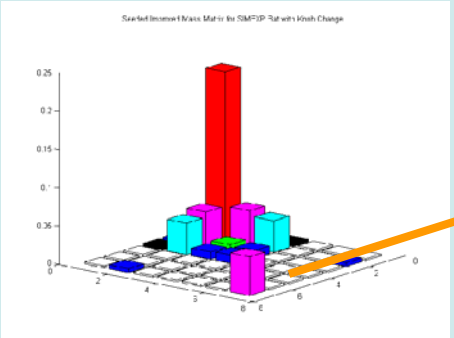
Analytical Bat Model

Simulated Experimental Model

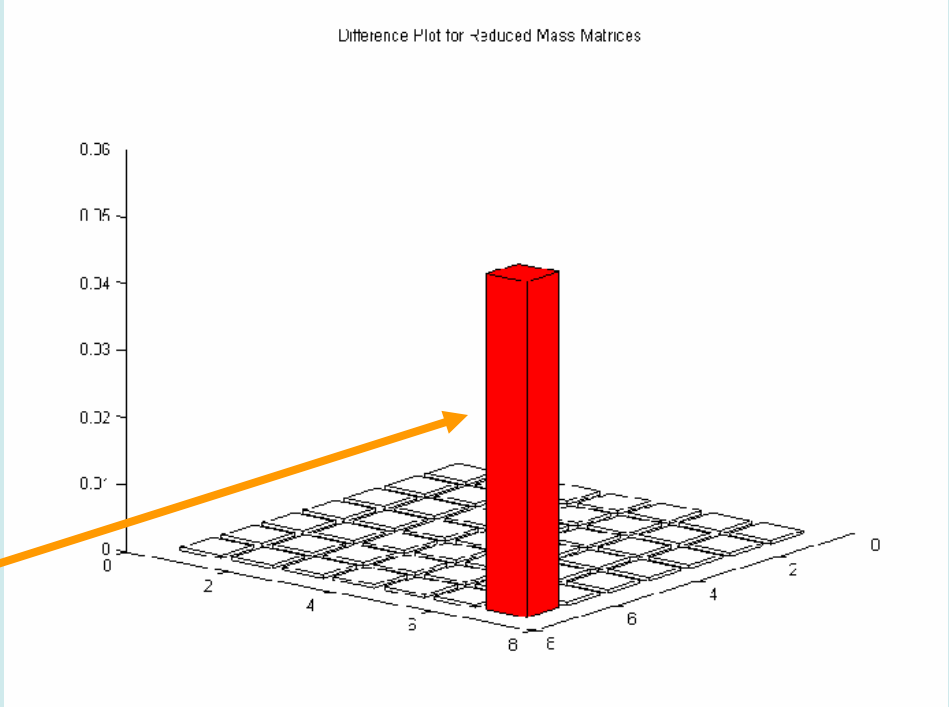
Resultant Mass Matrix Discrepancy through AMI



Reduced Analytical Mass Matrix



Simulated Experimental Mass Matrix



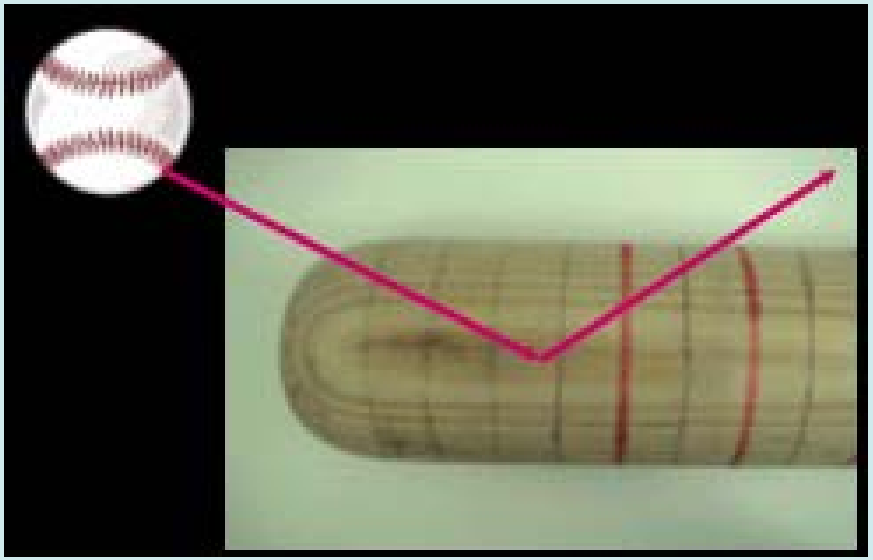
Mass Matrix Discrepancy

Conclusions

- The additional mass is easily identified through the Analytical Model Improvement (AMI) process
- Investigation indicates validity of proposed bat testing methodology

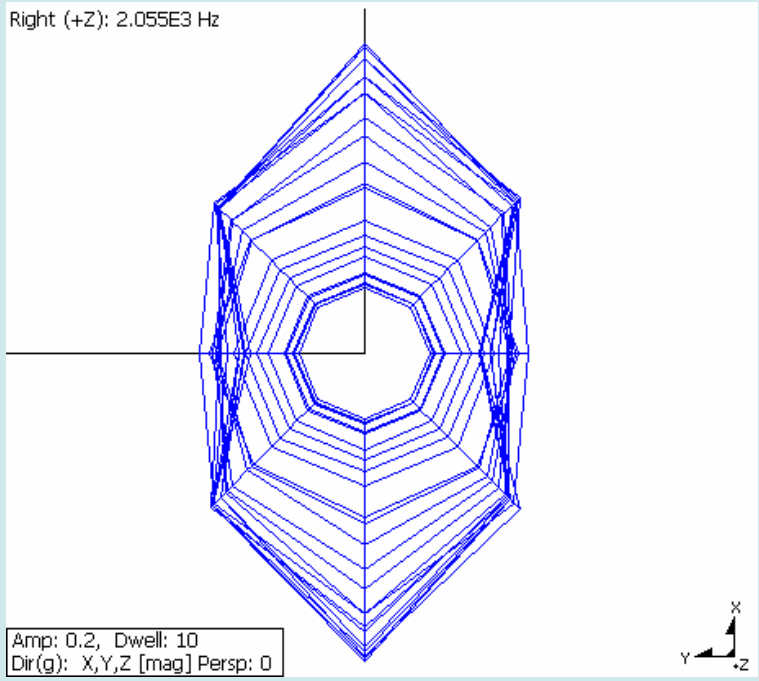
Future Work

- Test method with real experimental data to validate the process
- Analyze the sensitivity of the method to bat variations and alterations



Future Work--Extensions

- Aluminum bats are the safety concern motivating this project
- The shell-like behaviors of an aluminum bat may require modifications of this method



End view of hollow bat shell behavior

Acknowledgements

- Vibrant Technologies
  - MEScopeVES experimental modal analysis software
- The Mathworks, Inc.
  - MATLAB numerical analysis software
- Hibbitt, Karlsson and Sorensen, Inc.
  - ABAQUS finite element software

Questions?

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